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Quantum derivatives and high-frequency gain in semiconductor superlattices

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Abstract. We derive simple difference formulas describing a small-signal absorption of THz field in a semiconductor superlattice driven by a microwave pump. We give a transparent geometric interpretation of these formulas that allows a search of optimum conditions for the gain employing only a simple qualitative analysis. Our theoretical approach provides a powerful tool for finding the correspondence between quasistatic and dynamic regimes in ac-driven semiconductor superlattices.

Introduction

Semiconductor superlattices (SSL) have attracted growing attention in view of their unique electronic properties, which can be used for generation, amplification and detection of a high-frequency electromagnetic radiation [1]. Nonlinear transport properties of SSL allows a generation of THz radiation in conditions of negative differential conductance (NDC) [2]. However, the static NDC makes SSL unstable against formation of high-field electric domains [3]. These electric domains are believed to be destructive for the THz gain in SSLs. Currently the main focus is on the possibilities to overcome this drawback within the scheme of dc-biased SSL [4, 5]. However, schemes of THz superlatice devices with ac pump fields are also under discussion [6, 7]. A strong microwave field $E_p(t) = E_1 \cos(\omega_1 t)$ pumps the SSL and a desirable signal field $E_s(t) = E_2 \cos(\omega_2 t)$ has a higher frequency $\omega_2 > \omega_1$. Because for typical SSLs the characteristic scattering time τ at room temperature is of the order of 100 fs, an interaction of the microwave fields with the miniband electrons is quasistatic ($\omega_1\tau \ll 1$). Importantly, we have showed recently that such quasistatic pump field can completely suppress domains in SSLs [6]. Two distinct possibilities exist for the signal field: It can be also quasistatic ($\omega_2\tau \ll 1$) or it cannot be described within the quasistatic approach if $\omega_2\tau \gtrsim 1$. The later situation, i.e. $\omega_1\tau \ll 1$ but $\omega_2\tau \gtrsim 1$, can be called *semiquasistatic interaction*. Thus, the semiquasistatic approach is introduced to describe an amplification of THz field in SSL under the action of microwave field.

1. Quantum derivatives

Our main findings are following. We consider the response of miniband electrons to an action of the total electric field $E(t) = E_0 + E_1 \cos(\omega_1 t) + E_2 \cos(\omega_2 t)$, where E_0 is the dc bias and $E_s = E_2 \cos(\omega_2 t)$ ($\omega_2 = m\omega_1$, $m \in \mathbb{N}$) is the weak signal (probe) field. In the real devices E_s is a mode of the resonator tuned to a desirable THz frequency.

We define the dimensionless absorption of the weak ac probe field in SSL as $A(\omega_2) = 2\langle V(t)\cos(\omega_2 t)\rangle_t$, where $V(t)$ is the miniband electron velocity defined in the units of the maximal miniband velocity, averaging $\langle \ldots \rangle_t$ is performed over time. Starting from the exact formal solution of the Boltzmann transport equation, we represent the absorption A as a sum of three terms [6, 8]

$$A = A^{harm} + A^{coh} + A^{incoh}. \qquad (1)$$

Here A^{harm}, A^{coh} and A^{incoh} describe the absorption (gain) seeded by generation of harmonics, the parametric amplification of the probe field due to a coherent interaction of the pump and the probe fields, and the nonparametric absorption, correspondingly. The derivation of semiquasistatic formulas is based on the use of the asymptotic saddle-point method.

Using this method we have get the following results. The term A^{harm} is just the expression for mth in-phase harmonic of the time-dependent current through SSL

$$A^{harm} = 2\left\langle I(U_{dc} + U_{ac}\cos(\omega_1 t))\cos(m\omega_1 t)\right\rangle_t , \qquad (2)$$

where $U_{dc} = eLE_0$ is the dc voltage, $L = Nd$ is the length of SSL (d is the period of SSL and N is the number spatial periods), $U_{ac} = eLE_1$ is the amplitude of ac voltage created by the pump field across SSL, the current $I(t)$ is normalized to the maximal current in SSL, I_0, corresponding to the maximal miniband velocity. Note that A^{harm} gives the main contribution to the absorption of a weak probe [6]. The expression for A^{harm} is not specific for the semiquasistic

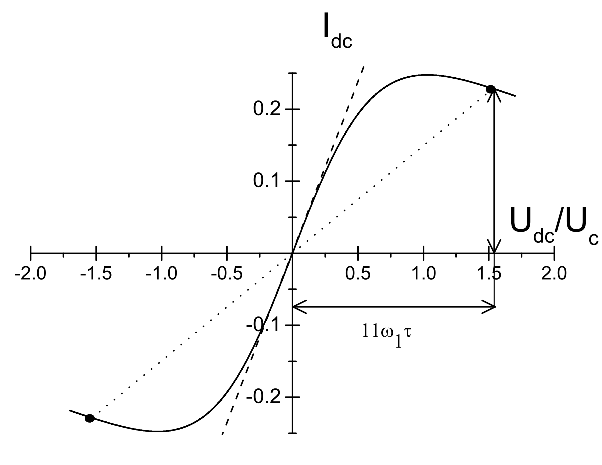

Fig. 1. Geometrical meaning of the incoherent component of absorption within semiquasistatic and quasistatic approaches. Time-averaged current I_{dc} under the action of quasistatic pump ($\omega_1\tau = 0.1$, $U_{ac}/U_c = 0.2$) vs dc voltage U_{dc}. If we choose working point at $U_{dc} = 0$, then the dotted segment corresponds to the finite difference for the weak probe at $\omega_2 = 11\omega_1$, the dashed straight line corresponds to the derivative. The slopes of these strait lines determines A^{incoh} in the semiquasistatic and quasistatic cases, respectively.

limit because of its independence on ω_2. However, *our main finding is that the absorption components A^{coh} and A^{incoh} can be represented within semiquasistatic approach using the specific terms of quantum derivatives as*

$$A^{coh} = eU_2 \left\langle \frac{I^{ET}\left(U_{dc} + U_{ac}\cos(\omega_1 t) + N\hbar\omega_2/e\right)}{2N\hbar\omega_2} \right.$$
$$\left. - \frac{I^{ET}\left(U_{dc} + U_{ac}\cos(\omega_1 t) - N\hbar\omega_2/e\right)}{2N\hbar\omega_2} \cos(2m\omega_1 t) \right\rangle_t , \quad (3)$$

(4)

$$A^{incoh} = eU_2 \left\langle \frac{I^{ET}\left(U_{dc} + U_{ac}\cos(\omega_1 t) + N\hbar\omega_2/e\right)}{2N\hbar\omega_2} \right.$$
$$\left. \frac{-I^{ET}\left(U_{dc} + U_{ac}\cos(\omega_1 t) - N\hbar\omega_2/e\right)}{2N\hbar\omega_2} \right\rangle_t , \quad (5)$$

where U_2 is the amplitude of small-signal voltage and

$$I^{ET}(U) = \frac{U/U_c}{1 + (U/U_c)^2} \quad (6)$$

is the Esaki-Tsu voltage-current (UI) characteristic ($U_c = \hbar N/e\tau$ is the critical voltage and I^{ET} is normalized to the maximal current $I_0 \equiv 2I^{ET}(U = U_c)$).

Importantly, following Eq. (5) in order to find the incoherent absorption in SSL at arbitrary high frequency ω_2 we need to know only

$$I_{dc}(U_{dc}) = \left\langle I^{ET}\left(U_{dc} + U_{ac}\cos(\omega_1 t)\right)\right\rangle_t , \quad (7)$$

that is, *the time-averaged current induced by the quasistatic field (voltage)*. For a given amplitude of ac voltage U_{ac}, the dc current I_{dc} is a function of only dc bias U_{dc}. It is easy to calculate or to measure the modifications of UI characteristic caused by the action of microwave (quasistatic) field [9, 10]. Note that in the quasistatic case A^{incoh} goes into a usual derivative of current. The difference between these derivatives is shown on the Figure 1 where the dashed line is a derivative of dc current and the dotted line is a quantum derivative depended on the number of harmonic m. The importance of finding of A^{incoh} is stipulated by the following fact: A^{incoh} plays an essential role in the stabilization of space-charge instability in SSL. We need to know A^{incoh} to determine the conditions of space-charge instabilities. It is surprise that we can determine it only measuring the dc current as a function a dc bias. Using only a finite difference we can select the working point of a generator that is optimal for its stability. Note that $A^{incoh} > 0$ in the quasistatic case and the system is stable against small fluctuations of internal field.

On the other hand, following Eq. (3) finding of the coherent component of absorption at the high frequency corresponding to mth harmonic of the pump frequency ($\omega_2 = m\omega_1$) is reduced to *the calculation of $2m$th harmonic of the current within quasistatic approach*. That is, one needs to know

$$V_m^{harm}(U_{dc}) = 2\left\langle I^{ET}(U_{dc} + U_{ac}\cos(\omega_1 t))\cos(m\omega_1 t)\right\rangle_t \quad (8)$$

(Note that in comparison with (2) we have in (8): $I(U) \to I^{ET}(U)$). In analogy with A^{incoh}, the coherent absorption A^{coh} can be found if we know only the dependence on dc bias. A^{coh} is proportional to the quantum derivative of A^{harm} in the semiquasistatic case, and A^{coh} goes into a common derivative in the quasistatic case. These derivatives are shown in the Figure 2 ($\langle V_m\rangle \equiv A^{harm}(m\omega_1)$)) for the case of 7 harmonic.

To the best of our knowledge this paper is the first work, where the quantum derivatives naturally appeared in a description of microstructure's response to a bichromatic field. It is the exciting problem to understand whether our semiquasistatic approach, developed for superlattices operating in the miniband transport regime, can be generalized to other tunnelling structure.

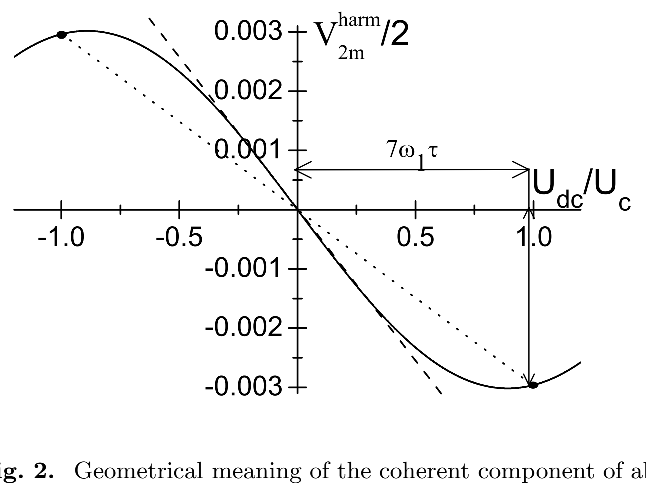

Fig. 2. Geometrical meaning of the coherent component of absorption witin semiquasistatic and quasistatic approaches. The $2m$th harmonic of quasistatic current V_{2m}^{harm} vs dc voltage U_{dc} for $m = 7$. The current is induced by the quasistatic pump ($\omega_1\tau = 0.14$) with the amplitude $U_{ac}/U_c = 8$. If we choose working point at $U_{dc} = 0$, then the dotted segment determines to the finite difference for the weak probe at $\omega_2 = 7\omega_1$, the dashed stright line corresponds to the derivative. The slopes of these lines determines A^{coh} within the semiquasistatic and quasistatic cases respectively.

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